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A boundary layer theory for the distribution of magnetisation in small spherical ferromagnetic particles

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Abstract. Boundary layer theory is applied to the micro-magnetic equations describing the distribution of magnetisation in small spherical ferromagnetic particles. A solution corresponding to a single domain wall naturally emerges. The spatial variation of magnetisation so obtained compares well with previous numerical treatments.

1. Introduction

The modern theory of ferromagnetic domain structure considers a magnetised body to be subdivided into domains; in each domain the magnetisation is assumed to be spatially uniform and the domains separated by domain walls. It is assumed that these walls have an energy per unit area arising from exchange and magnetic anisotropy forces. The equilibrium domain structure is the one that minimises the total energy, which has contributions from the wall energy, magneto-anisotropy energy and the magnetostatic energy. As Fuller Brown has emphasised, this procedure for obtaining possible domains structures is very dependent on the ingenuity of the proposer of the structure. Even so it is an extremely valuable way for studying ferromagnetic materials and has been used extensively (see for example Craik and Tebble (1970)). By far the most difficult contribution to evaluate is the magnetostatic energy, and this has meant that the domain structures that have been studied have been chosen such that this evaluation is relatively straightforward.

An entirely different but more fundamental approach comes under the heading of micro-magnetics. Here the total energy is expressed as an integral over the whole body of a function of the local direction of magnetisation. (The magnitude is usually assumed constant.) The extremal of the energy with regard to arbitrary variations of this function gives rise to Euler equations for this function in the form of nonlinear differential integral equations. In this method too, the magnetostatic contribution is the most difficult to include, and the majority of previous analytic methods have been based on particular classes of solution where the magnetostatic contribution is relatively straightforward. For example, the magnetisation is assumed to 'curl' so as to avoid the occurrence of a magnetostatic energy contribution. A notable exception is the work of Eisenstein and Aharoni (1976). These authors have used a variational method to study the micro-magnetic structure in a spherical particle. They treat the magnetostatic energy contribution to the same approximation as the other energy contributions. However, the energy of their assumed class of trial functions for the distribution is always greater than that obtained numerically by Stapper (1968). Hence, in the spirit of

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a variational calculation, the magnetic distribution discussed by Stapper must be preferred and it is the one that is discussed in this paper. For a review of the subject of micro-magnetics, see for example Brown (1963).

Both Brown and Labonte (1965) and Stapper (1969) have solved the full micro-magnetic equations numerically. It is apparent from these numerical results that for particles which are not too small, the magnetisation is almost uniform over a substantial part of the volume of the body, though not necessarily in the direction of easy magnetisation. The rapid change in magnetisation that does take place is over a relatively small region. If one identifies this latter region as a domain wall, then these numerical results illustrate how the micro-magnetic approach can lead to a macroscopic domain theory, as outlined in the first paragraph of this Introduction.

The purpose of this paper is to study analytically the problem solved numerically by Stapper. The analysis is based on the observation, mentioned above, that if there are regions where the magnetisation changes rapidly in direction (domain walls) surrounded by regions where the changes are very much less (domains), then the wall regions may be treated as boundary layers. In other branches of physics the boundary layer concept is well accepted, but to the author's knowledge the idea of using such a concept to treat the micro-magnetic equations is new. The particular problem studied by Stapper is that of a spherical particle in which it is assumed that the direction of magnetisation always remains parallel to a plane, but changes as a function of the perpendicular distance from this plane. This assumption reduces the micro-magnetic equation to a nonlinear integro-differential equation in one variable. Though this assumption may be too restrictive to apply to the real situation, it is sufficiently general to allow one to study the possibility of treating domain walls as boundary layers. A significant advantage of the present analytic treatment over Stapper's numerical one is that the scaling of the changes with respect to the various magnetic parameters is given explicitly.

2. Basic micro-magnetic equations

We consider a spherical particle of radius a in which the magnetisation $\boldsymbol{M}$ at any point is given by

$$\boldsymbol{M} = M_0(\cos\phi, \sin\phi, 0)$$

where ϕ is a function of one variable only, namely z. M_0 is an absolute constant.

The magnetic anisotropy is uniaxial with the easy direction in the x direction. The total magnetic anisotropy energy is then

$$E_A = \pi K \int_{-a}^{+a} (a^2 - z^2)\sin^2\phi \, dz$$

where K is the anisotropy constant. The exchange energy is given by

$$E_0 = \alpha\pi \int_{-a}^{+a} (\alpha^2 - z^2)\left(\frac{d\phi}{dz}\right)^2 dz,$$

where α is the exchange energy current. The magnetostatic contribution is by far the most complicated, and as shown in appendix 1 may be written in the form

$$E_{\mathrm{m}} = M_0^2 \int_{-a}^{+a} \mathrm{d}z \int_{-a}^{+a} \mathrm{d}z' \, H(z, z') \cos[\phi(z) - \phi(z')]. \tag{2.1}$$

Magnetostrictive effects are neglected.

The micro-magnetic equation is obtained by demanding that the total energy, $E_{\mathrm{T}} = E_{\mathrm{A}} + E_0 + E_{\mathrm{m}}$, is an extremum with respect to arbitrary variations of ϕ. This leads to the Euler equation, which in dimensional form is

$$\varepsilon^2 \left(\frac{\mathrm{d}^2\phi}{\mathrm{d}y^2} - \frac{2y}{(1-y^2)} \frac{\mathrm{d}\phi}{\mathrm{d}y} \right) = \sin\phi \cos\phi - \eta \int_{-1}^{+1} \sin[\phi(y) - \phi(y')] \bar{H}(y, y') \, \mathrm{d}y' \tag{2.2}$$

where $y = z/a$, $\eta = M_0^2/K$, $\varepsilon = \delta/a$ and $\delta = (\alpha/K)^{1/2}$. Expressions for

$$\bar{H}(y, y') \equiv H(y, y')/[a\pi(1-y^2)],$$

are given in appendix 1. It may be noted that a solution of (2.2) is $\phi \equiv 0$; this corresponds to the uniformly magnetised state.

3. Boundary layer theory

The boundary layer theory to be presented in this section is based on the assumption that $\varepsilon \ll 1$. In the usual theory of domain walls in which magnetostatic effects are neglected ($\eta \equiv 0$), one finds that δ is a measure of the width of such walls (see for example Craik and Tebble (1970)). Thus the condition $\varepsilon (\equiv \delta/a) \ll 1$ implies that the theory is limited to particles whose size is significantly larger than the intrinsic width of a domain wall. We further restrict our attention to the case where only one wall is present, though the analysis may readily be extended. This wall is centred about $y = 0$ as this position obviously minimises the magnetostatic energy. We further impose the condition that $\phi \to 0$ as $y \to -1$ and $\phi \to \pi$ as $y \to +1$. This configuration coincides with one of those studied numerically by Stapper.

To apply boundary layer theory to (2.2), we treat ε as small and follow standard treatments by making inner and outer expansions (see for example Nayfeh (1973, ch 6)). For the inner expansion about $y = 0$, where we assume that the main variation of ϕ occurs, we introduce the new coordinate $\xi = y/\varepsilon$ and consider the limit $\varepsilon \to 0$, with ξ remaining finite. Equation (2.2) reduces to

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2} = \sin\phi \cos\phi - \eta \int_{-1}^{+1} \sin[\phi(\xi) - \phi(y')] \bar{H}(0, y') \, \mathrm{d}y'.$$

It is essential to retain the variable y' under the integral, and in this respect the present treatment differs from boundary layer theory applied to differential equations. Writing $\phi = \pi/2 + \psi$ and assuming the symmetry $\psi(\xi) = -\psi(-\xi)$, and noting that $\bar{H}(0, y) = \bar{H}(0, -y)$, the above equation reduces to

$$\mathrm{d}^2\psi/\mathrm{d}\xi^2 = -\sin\psi \cos\psi + A \sin\psi, \tag{3.1}$$

where

$$A = -\eta \int_{-1}^{+1} \cos[\psi(y)] \bar{H}(0, y) \, \mathrm{d}y. \tag{3.2}$$

This equation may be studied using phase-plane arguments (Minorsky 1947) or integrated in terms of elliptic functions. Either way one finds that periodic solutions for ψ exist, but for one choice of the integration constants the period becomes infinite. For this case

$$(\mathrm{d}\psi/\mathrm{d}\xi)^2 = (\cos\psi - A)^2,$$

which may be integrated to give, for $|A|<1$,

$$\psi = 2\tan^{-1}\left[\left(\frac{1-A}{1+A}\right)^{1/2}\tanh\theta\right], \tag{3.3}$$

where $\theta = \frac{1}{2}\xi[(1-A^2)]^{1/2}$. This solution corresponds to a solitary pulse or soliton. It has less energy than the periodic solutions, and hence is the most probable state.

$$\text{For } \xi \to \pm\infty, \qquad \psi \to \pm(\pi/2 - \sin^{-1} A).$$

To complete the solution, one must satisfy the consistency condition (3.2) which, using the form for ψ given by (3.3), takes the form

$$A = \frac{-\eta(1-A^2)}{(1+\beta\eta)}\int_{-1}^{-1}\frac{\bar{H}(0,y)\,\mathrm{d}y}{A+\cosh(2\theta)}, \tag{3.4}$$

where $\beta = \int_{-1}^{+1}\bar{H}(0,y)\,\mathrm{d}y$. The integrals are evaluated in appendix 2, and it is found that for small ε

$$A \approx \frac{4\eta\varepsilon\ln\varepsilon\cos^{-1}A}{(1+4\pi\eta/3)} + \eta\mathrm{O}(\varepsilon).$$

For small A this may be solved to give

$$A \approx 2\pi\eta\varepsilon\ln\varepsilon/(1+4\pi\eta/3). \tag{3.5}$$

Though in the above analysis we have treated A as being independent of ε (at least as existing in the limit as $\varepsilon \to 0$), and have now found that $A \approx \varepsilon\ln\varepsilon$, this is not inconsistent since the terms we have neglected in the limiting procedure are at least of order ε.

To complete the solution one now considers an outer expansion appropriate to finite values of y. We have seen that the inner expansion leads to an assymptotic value of $\psi = \pm(\pi/2) - \sin^{-1}A)$, and since $A \approx \varepsilon\ln\varepsilon$ and $\varepsilon \ll 1$ we expect the value of ψ in the outer region to be of order $\pm\pi/2$. Thus in this region we write $\psi = \pm(\pi/2 + \delta\psi)$ and linearise the equation for ψ. However, in evaluating the integral that appears in (2.2), it is necessary to consider ψ for all y. Thus we divide the integral into inner and outer regions; in the inner region of width $2y_0$ we replace ψ by the expression given by (3.3), and in the outer region we use the linearised form mentioned above. In this way we obtain the following linearised equation for $\delta\psi$, for $y > y_0$:

$$\varepsilon^2\left(\frac{\mathrm{d}^2\delta\psi}{\mathrm{d}y^2} - \frac{2y}{(1-y^2)}\frac{\mathrm{d}\delta\psi}{\mathrm{d}y}\right)$$

$$= \delta\psi - \eta\left[\delta\psi\left(\int_{y_0}^{1}[\bar{H}(y,y') - \bar{H}(y,-y')]\,\mathrm{d}y' + \int_{-y_0}^{y_0}\bar{H}(y,y')\sin[\psi(y')]\,\mathrm{d}y'\right)\right]$$

$$+ \eta\int_{y_0}^{1}[\bar{H}(y,y') + \bar{H}(y,-y')]\delta\psi(y')\,\mathrm{d}y' - \eta\int_{-y_0}^{y_0}\bar{H}(y,y')\cos[\psi(y')]\,\mathrm{d}y'.$$

We further assume that $\delta\psi$ is slowly varying for $y > y_0$, and hence we may take $\delta\psi$ outside the integral. Then by considering the limit $\varepsilon \to 0$ ($y_0 \simeq \varepsilon$) in all terms except the last term on the right-hand side of the above equation, we find

$$[1+\eta\lambda(y)]\delta\psi(y) = \eta \int_{-y_0}^{y_0} \bar{H}(y, y') \cos[\psi(y')]\, dy' \equiv S(y),$$

where

$$\lambda(y) = 2\int_0^{+1} \bar{H}(y, -y')\, dy'. \tag{3.6}$$

The limiting procedure used above is seen to be consistent, since the terms neglected will only change the coefficient of $\delta\psi$ by order ε compared to unity, whereas proceeding to the limit $\varepsilon = 0$ in the term now denoted by $S(y)$ would imply $\delta\psi = 0$. We may write, since $\cos\psi \to A$ for $y > \varepsilon$,

$$\frac{S(y)}{\eta} = \int_{-1}^{+1} \bar{H}(y, y') \cos[\psi(y')]\, dy' - A\int_{y_0}^{1} [\bar{H}(y, y') + \bar{H}(y, -y')]\, dy',$$

but in the last integral we may proceed to the limit $\varepsilon = 0$ to give

$$\frac{S(y)}{\eta} = \int_{-1}^{+1} \bar{H}(y, y') \cos[\psi(y')]\, dy' - AK(y), \tag{3.7}$$

where

$$K(y) = \int_0^1 dy'[\bar{H}(y, y') + \bar{H}(y, -y')]. \tag{3.8}$$

In appendix 2 it is shown that $K(y) \equiv 4\pi/3$ and $\lambda(0) = 4\pi/3$. Further using the definition of A as given by (3.2), we see that if we extrapolate the above form for $\delta\psi(y)$ to $y = 0$ that

$$\delta\psi(0) = -A\frac{[1+\eta K(0)]}{1+\eta\lambda(0)} = -A.$$

We may now obtain a solution for ψ for all y by forming a *composite* solution of the solutions obtained in the inner and outer regions. However, since for $\xi(=y/\varepsilon) \to \infty$ the inner solution is such that $\psi \to \pi/2 - \sin^{-1} A$, whilst for $y \to 0$ the outer solution $\psi \to \pi/2 - A$, which gives continuity at least for small A, it is best for arbitrary A to consider a composite solution for the direction cosine $\cos\psi$.

Thus we write ($y = z/a$)

$$\cos\psi = \frac{(1-A^2)}{A + \cosh[(1-A^2)^{1/2} y/\varepsilon]} - \frac{S(y)}{[1+\eta\lambda(y)]}, \tag{3.9}$$

and take this to be valid for all $y \geq 0$. It is shown in appendix 2, that, to sufficient accuracy, we may write

$$S(y) \simeq 2\varepsilon\eta \cos^{-1}(A)\bar{H}(y+\varepsilon, 0),$$

and

$$\lambda(y) \simeq \tfrac{4}{3}\pi(1 - 3y/4).$$

The above solution has all the qualitative features of the numerical solution obtained by Stapper. In particular, a region exists (domain wall) where the magnetisation changes rapidly. From the above it is seen that the characteristic length of this variation is $\delta/(1-A^2)^{1/2}$. Outside this region the magnetisation direction slowly relaxes to the magnetic easy directions ($\psi = \pm\pi/2$), The characteristic length associated with this variation being a, the radius of the particle. The maximum excursion away from the easy direction in this region is given by $\cos\psi = A$. For the parameters considered by Stapper ($\eta = 0.47$, $\varepsilon = 0.065$), equation (3.5) gives $A \simeq -0.8$, which corresponds to a 10^0 excursion. This seems to be in excellent agreement with the value obtained by Stapper, who unfortunately only presents his results graphically.

4. Conclusions

Boundary layer theory has been applied to the micro-magnetic equations describing the distribution of magnetisation in a spherical ferromagnetic particle. An explicit analytic solution has been obtained showing how this distribution depends on the magnetic properties and the size of the particle. In agreement with earlier numerical solutions, the analytic one clearly shows the existence of skirts, where the magnetisation direction slowly relaxes to the magnetic easy directions.

The method is readily extended to the case where there is more than one wall per particle. To the same approximation as used in this paper, each domain wall may be treated separately; the only difference arises in the evaluation of A. In the consistency condition for A of the form (3.2), $\bar{H}(0, y)$ must be replaced by $\bar{H}(y_c, y)$ where y_c is the position of the centre of the domain wall. The equation for ψ is still of the form (3.1), but now with $\xi = (y-y_c)/\varepsilon$. The calculation $\delta\psi$ proceeds as in this paper, but now $S(y) \propto \bar{H}(y-y_c+\varepsilon, 0)$. With contributions to $S(y)$ coming from each domain wall region, this is again in qualitative agreement with the solutions shown graphically in Stapper (1968).

Appendix 1

By introducing the idea of magnetic poles (see for example Rhodes *et al* (1962)), it may be seen that the magnetostatic energy of the spherical particle considered in the text may be written as the sum of terms, each term corresponding to the magnetostatic energy of two discs with pole distributions proportional to $\cos\phi\sin\theta$. However, from the axial symmetry this energy is proportional to

$$\sin\theta\sin\theta'(\cos\phi\cos\phi' + \sin\phi\sin\phi')H(z, z').$$

Here θ, ϕ are the angles in a spherical polar coordinate system such that the rotation of the magnetisation vector is due to a change of ϕ, where θ' is the value of θ at $z = z'$. Here $H(z, z')$ is the energy of the discs when their magnetisation vectors are parallel. Remembering that an element of surface is just $\mathrm{d}z/\sin\theta$, the total magnetostatic energy of the spherical particle is as given by (2.1). From elementary magnetostatics we may write

$$H(z, z') = rr' \int_0^{2\pi} \mathrm{d}\theta \int_0^{2\pi} \frac{\mathrm{d}\theta' \cos\theta\cos\theta'}{[r^2 + (r')^2 - 2rr'\cos(\theta-\theta') + (z-z')^2]^{1/2}},$$

where r, r' are the radii of the discs. For spherical particles we have $r^2 = a^2 - z^2$. This may be integrated once to give

$$H = rr'\pi \int_0^{2\pi} \frac{\cos\theta \, d\theta}{[r^2 + (r')^2 - 2rr'\cos\theta + (z - z')^2]^{1/2}},$$

and integrated again to give

$$H = 2\pi(rr')^{1/2}[(2b - 1/b)K(1/b) - 2bE(1/b)]$$

where K and E are the complete elliptic integrals and

$$b^2 = \frac{(r + r')^2 + (z - z')^2}{4rr'} = \frac{1 - yy' + (1 - y^2)^{1/2}(1 - y'^2)^{1/2}}{2(1 - y^2)^{1/2}(1 - y'^2)^{1/2}}.$$

In particular we have $(z/a = y)$

$$\bar{H}(y, 0) = \frac{H(y, 0)}{a\pi(1 - y^2)} = \frac{2}{(1 - y^2)^{1/2}} F(b_0)$$

where $b_0 = [1 + (1 - y^2)]^{1/2}/2(1 - y^2)^{1/2}$ and

$$F(b) = (2b - 1/b)K(1/b) - 2bE(1/b).$$

For $y \to 1$ we find $\bar{H}(y, 0) \approx \frac{1}{4}\pi\sqrt{2}(1 - y^2)^{1/4}$ and for $y \to 0$ $\bar{H}(y, 0) \approx 2\ln(8/y)$. Also $\bar{H}(0, y) \approx 2\ln(8/y)$ for $y \to 0$.

A different representation for H may be obtained as follows. First let $z/a = \cos\phi$, $z'/a = \cos\phi'$ and $r = a\sin\phi$ in which case

$$H = \frac{a\pi}{\sqrt{2}} \sin\phi \sin\phi' \int_0^{2\pi} \frac{\cos\eta \, d\eta}{(1 - \cos\gamma)^{1/2}},$$

where $\cos\gamma = \cos\phi\cos\phi' - \sin\phi\sin\phi'\cos\eta$. Then, using the properties of Legendre functions, namely

$$\frac{1}{(1 - \cos\gamma)^{1/2}} = \sqrt{2} \sum_{n=0}^{\infty} P_n(\cos\gamma),$$

$$P_n(\cos\gamma) = P_n(\cos\phi)P_n(\cos\phi') + 2\sum_{m=1}^{\infty} \frac{(n - m)!}{(n + m)!} P_n^m(\cos\phi)P_n^m(\cos\phi')\cos(m\eta)$$

and

$$P_n^1(\cos\phi) = \sin\phi P_n'(\cos\phi),$$

where the prime on the P_n' denotes differentiation with respect to the argument, we find

$$H(z, z') = 2a\pi^2 \sin^2\phi \sin^2\phi' \sum_{n=1}^{\infty} \frac{P_n'(\cos\phi)P_n'(\cos\phi')}{n(n + 1)}.$$

Thus

$$\bar{H}(y, y') = 2\pi[1 - (y')^2] \sum_{n=1}^{\infty} \frac{P_n'(y)P_n'(y')}{n(n + 1)}.$$

Appendix 2

To evaluate the quantity A given by (3.4) it is necessary to consider the following integral:

$$I=\int_{-1}^{+1}\frac{\bar{H}(0,y)\,\mathrm{d}y}{A+\cosh(2\phi)}.$$

Since $2\phi=(1-A^2)^{1/2}y/\varepsilon$ and $\varepsilon \ll 1$, the major contribution to the integral comes from small y. Thus we may replace $\bar{H}(0,y)$ by its limit as $y\to 0$, which is given in appendix 1. Then to lowest significant order

$$I=-\frac{4\varepsilon\ln\varepsilon}{(1-A^2)^{1/2}}\int_0^\infty\frac{\mathrm{d}x}{A+\cosh x}+\mathrm{O}(\varepsilon^2)$$
$$\approx -(4\varepsilon\ln\varepsilon\cos^{-1}A)/(1-A^2).$$

The remaining integrations required in the text are best performed by using the Legendre representation of $\bar{H}$. In particular, consider $\lambda(y)$ defined by (3.5). With the representation of $\bar{H}$ given in appendix 1 we have

$$\lambda(y)=4\pi\sum_{n=1}\frac{P_n'(y)}{n(n+1)}\int_0^1 P_n'(-y)(1-y^2)\,\mathrm{d}y.$$

Using the properties of Legendre functions, the integrals may be evaluated to give

$$\lambda(y)=\frac{4\pi}{3}\left(1+3\sum_{m=1}^{\infty}\frac{P_{2m}'(y)P_{2m}(0)}{(2m+2)(2m-1)}\right).$$

In particular $\lambda(0)=4\pi/3$. In a similar manner it may be shown that $K(y)=4\pi/3$.

The variation of $\lambda(y)$ with y is smooth, and thus it is sufficient for most purposes to obtain a simple analytic approximation. For $y=1$ the summation may be carried out and one finds $\lambda(1)=(4\pi/3)(2-5\sqrt{2}/4)\approx(4\pi/3)/4$. Thus we write for $0<y<1$

$$\lambda(y)\approx\tfrac{4}{3}\pi(1-3y/4).$$

Using the definition of $S(y)$ as given by (3.6), and the form of ψ as given by (3.3), we have

$$S(y)=\eta(1-A^2)\int_{-1}^{+1}\frac{\bar{H}(y,y')\,\mathrm{d}y'}{A+\cosh(2\theta)}.$$

The major contribution to this integral comes from small y', and thus for $y\gg\varepsilon$ we have

$$S(y)=2\varepsilon\eta\bar{H}(y,0)\cos^{-1}A.$$

A closed analytic expression for $\bar{H}(y,0)$ is given in appendix 1. Comparison of the above integral definition of $S(y)$, for $y=0$, and the consistency equation for A, namely (3.4), shows that $S(0)\approx -4\eta\varepsilon\ln\varepsilon\cos^{-1}A$. Since in this limit $\bar{H}(y,0)\approx -2\ln y$, if we write

$$S(y)\simeq 2\varepsilon\eta\bar{H}(y+\varepsilon,0)\cos^{-1}A,$$

we have a form which has the correct form for $y<\varepsilon$ and $y>\varepsilon$.

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